Section 1.1 The algebra of Complex Numbers

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Section 1.1 The algebra of Complex Numbers

If we attempt to solve the equation $x^2 + 1 = 0,$

Obviously, we don't have a real solution.

But if we insist on having a solution, we should then have

$x^2 = -1$ or $x = \sqrt{-1}.$

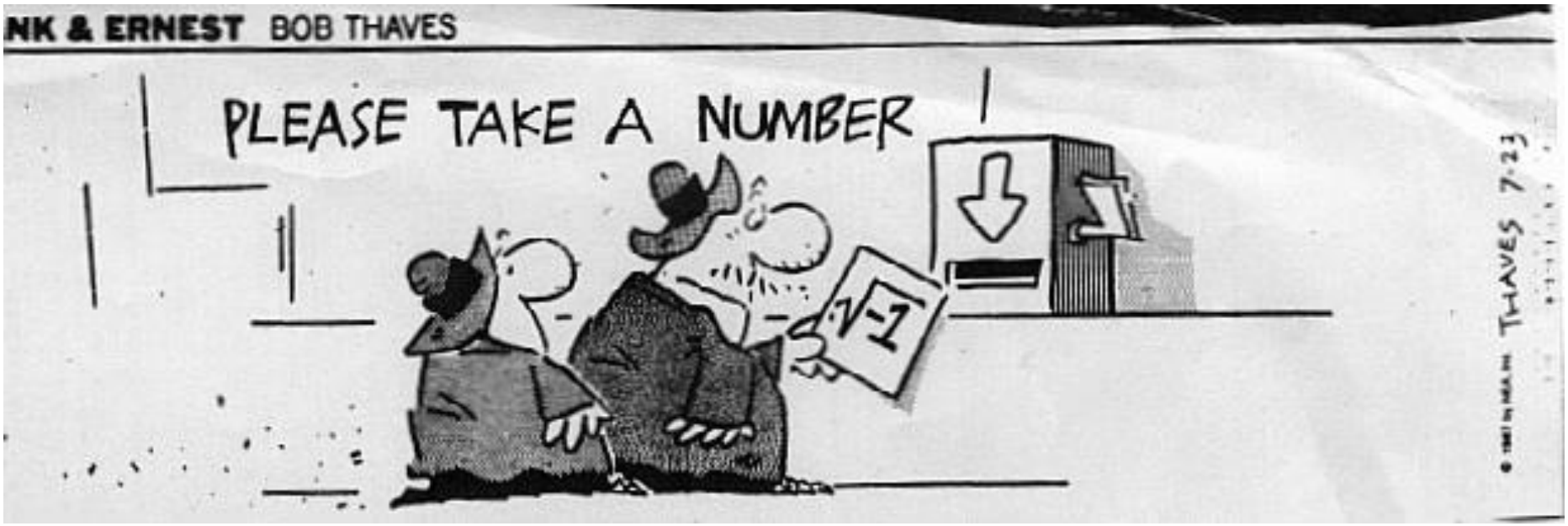
$\sqrt{-1} = i$

Instead of $\sqrt{-1},$ we will use the symbol i.

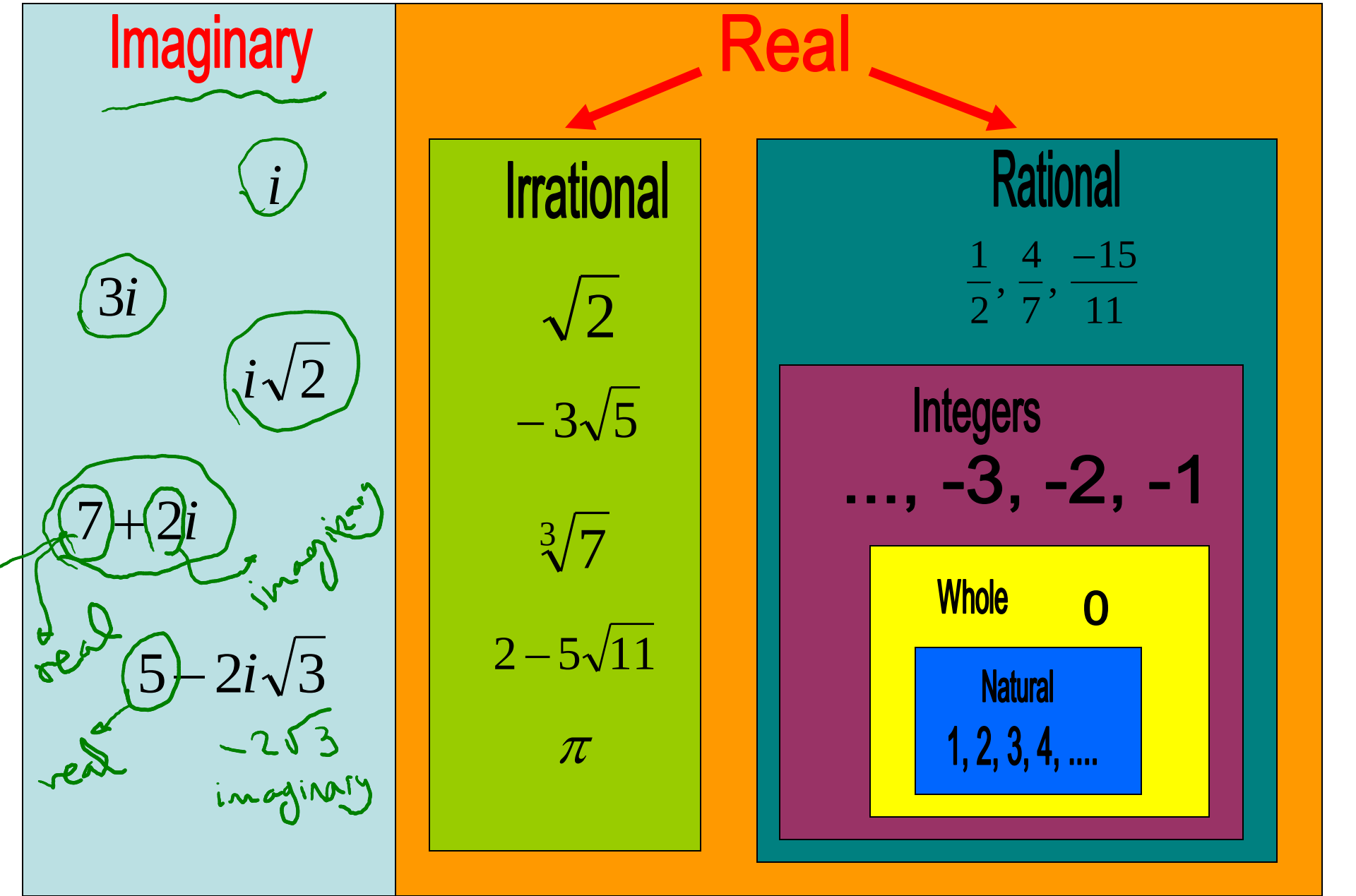
So by definition $i = \sqrt{-1},$ and we call it the imaginary unit.

$i^2 = -1$

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Complex Number System



Property of the square root of negative numbers

- If r is a **positive** real number, then

$$\sqrt{-r} = i\sqrt{r}$$

Examples:

$\sqrt{-3} = i\sqrt{3}$

$\sqrt{-4} = i\sqrt{4} = 2i$

$\sqrt{3}\sqrt{-1} = \sqrt{3}\,i$

$\sqrt{4}\sqrt{-1} = 2i$

Complex Numbers

- A complex number has a real part & an imaginary part.
- Standard form is:

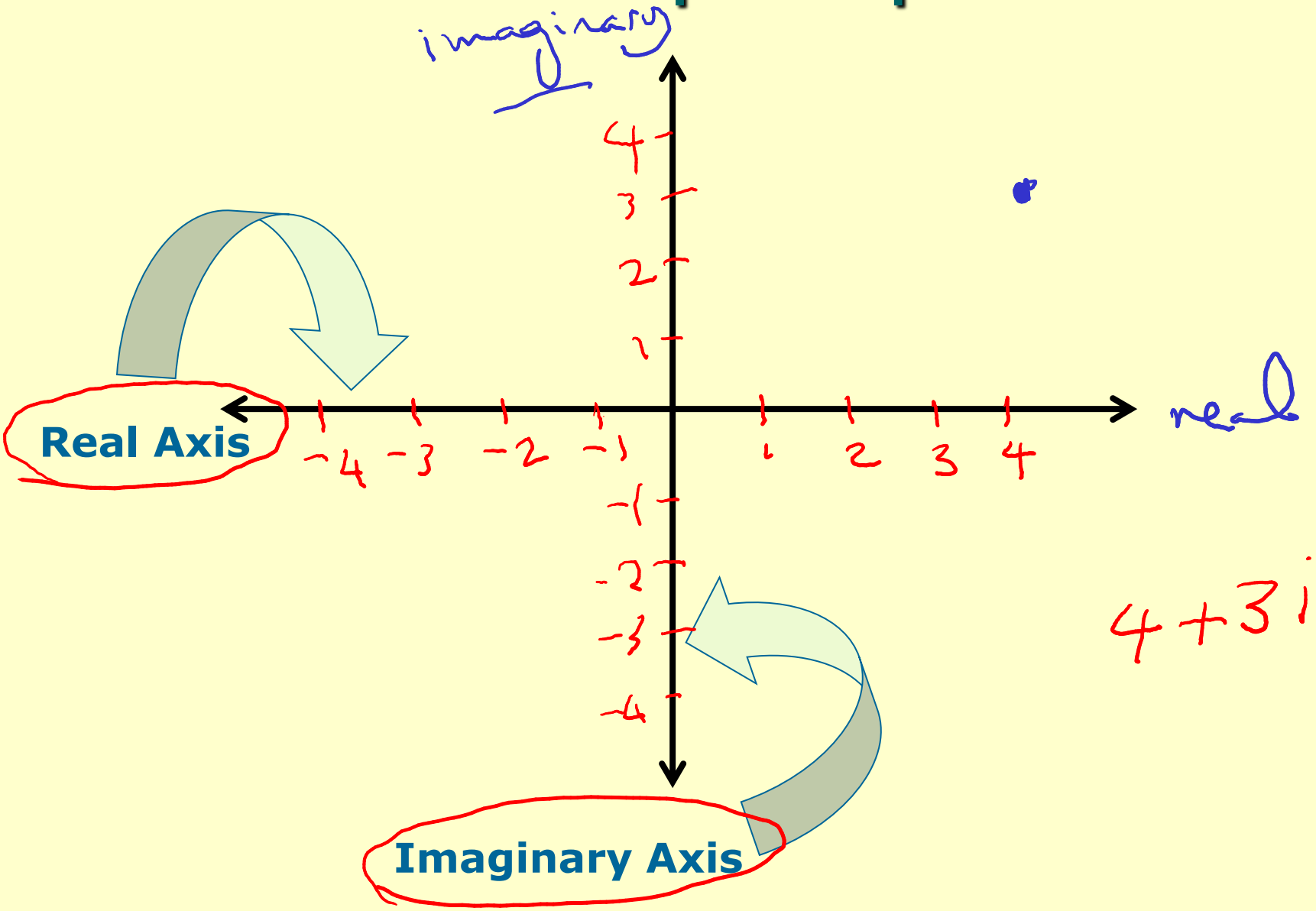
$$a+bi$$

Real part

Imaginary part

Example: 5+4i

The Complex plane



Adding and Subtracting (add or subtract the real parts, then add or subtract the imaginary parts)

Ex: $(-1+2i)+(3+3i)$

$=(-1+3)+(2+3)i$

$=2+5i$

Ex: $2i-(3+i)+(2-3i)$

$=(0-3+2)+(2-1-3)i$

$=-1-2i$

$2i-3-i+2-3i$

$=-1-2i$

Ex: $(2-3i)-(3-7i)$

$=(2-3)+[-3-(-7)]i$

$=-1+4i$

$2-3i-3+7i$

$=-1+4i$

Multiplying
Treat the i's like variables, then change any i^2 to -1

Ex: $-i(3+i)$

$= -3i - i^2$

$= -3i - (-1)$

$= -3i + 1$

$= 1 - 3i$

Ex: $(2+3i)(-6-2i)$

$= -12 - 4i - 18i - 6i^2$

$= -12 - 22i - 6(-1)$ $+6$

$= -12 - 22i + 6$

$= -6 - 22i$

Division of complex numbers

We use the aspect of complex conjugates in order to divide two complex numbers

Consider $\dfrac{2 + 9i}{5 - 2i} \times \dfrac{5+2i}{5+2i}$

We can rewrite this as $\dfrac{(2 + 9i)(5+2i)}{(5 - 2i)(5+2i)}$

Operations on Complex N°s

- *Division of complex numbers*

hence

$$\frac{(2 + 9i)(5+2i)}{(5 - 2i)(5+2i)} = \frac{10 - 18 + 49i}{25 + 4} = -\frac{8}{29} + \frac{49}{29}i$$

$10 + 4i + 45i + 18i^2$

$10 + 4i + 45i - 18$

$25 - 4i^2$

$25 - 4(-1)$

$25 + 4$

$= 29$

$$Ex: \frac{3+11i}{-1-2i} \times \frac{-1+2i}{-1+2i}$$

$$= \frac{(3+11i)(-1+2i)}{(-1-2i)(-1+2i)}$$

$$= \frac{-3+6i-11i+22i^2}{1-2i+2i-4i^2}$$

$$= \frac{-3-5i+22(-1)}{1-4(-1)}$$

$$= \frac{-3-5i-22}{1+4}$$

$$= \frac{-25-5i}{5}$$

$$= \frac{-25}{5} - \frac{5i}{5}$$

$$= -5-i$$

Solved Problems on Section 1.1

Problem 1



Solve each of the following equations for z

$$(a)\ \frac{z}{1-z}=1-5i \ \Rightarrow\ z=(1-5i)(1-z)\ \Rightarrow$$

$$z=1-5i-z(1-5i)\ \Rightarrow\ z+z(1-5i)=1-5i\ \Rightarrow$$

$$(2-5i)z=1-5i\ \Rightarrow\ z=\frac{1-5i}{2-5i}=\frac{27}{29}-\frac{5}{29}i$$

$$(b)\ z^2+16=0\ \Rightarrow\ z^2-(-16)=0\ \Rightarrow\ z^2-(4i)^2=0$$

$$\Rightarrow\ (z+4i)(z-4i)=0\ \Rightarrow\ z=\pm 4i.$$

Solved Problems on Section 1.1

Problem 2

Find all solutions of the equation $z^4 - 16 = 0.$

Solution

$$z^4 - 16 = (z^2 - 4)(z^2 + 4)$$

$$= (z + 2)(z - 2)(z + 2i)(z - 2i)$$

So, the solutions of the equation are

$$z = \pm 2,\ z = \pm 2i.$$

$z+2=0$ $z=-2$

$z-2=0$ $z=2$

$z+2i=0$ $z=-2i$

$z-2i=0$ $z=2i$

Solved Problems on Section 1.1

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