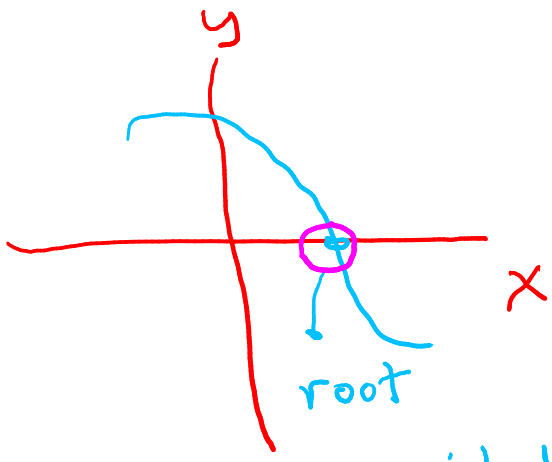
Chapter 5

Root appoximation



تقاطع المنحنى مع محور x

$$f(x) = x^2 - 2x - 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1)$$

$$x = 3 \qquad x = -1$$

$$f(x) = x^{1.5} - x \ln x$$

Bisection method

The Bisection Method

Step 1: Choose lower x_l and upper x_u guesses for the root such that the function changes sign over the interval. This can be checked by ensuring that $f(x_l)f(x_u) < 0$.

دالة سالبة ودالة موجبة

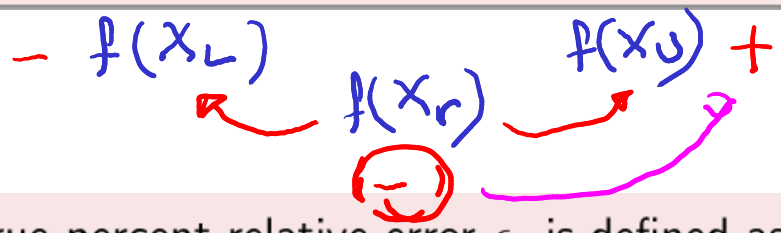
Step 2: An estimate of the root x_r is determined by: $x_r = \frac{x_l + x_u}{2}$

Step 3: Make the following evaluations to determine in which subinterval the root lies:

(a) If $f(x_l)f(x_r) < 0$, the root lies in the lower subinterval. Therefore, set $x_u = x_r$ and return to step 2.

(b) If $f(x_l)f(x_r) > 0$, the root lies in the upper subinterval. Therefore, set $x_l = x_r$ and return to step 2.

(c) If $f(x_l)f(x_r) = 0$, the root equals x_r, terminate the computation.



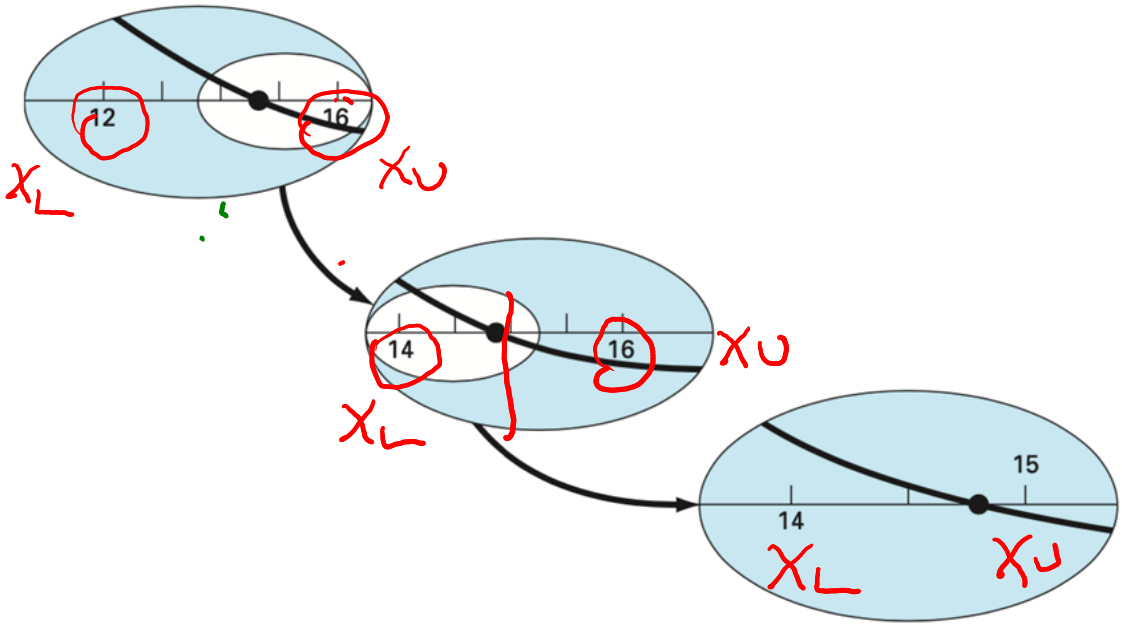
The true percent relative error ϵ_t is defined as

$$\epsilon_t = \left| \frac{\text{true value} - \text{approximation}}{\text{true value}} \right| \times 100\%$$

معطى

bisection

5.2 The Bisection Method



Question 1: Use the bisection and false-position methods to determine the real root of the function

$f(x) = 0.1x^2 - x \ln x$

in the interval $[0.8, 1.6]$. Iterate until the estimated error ϵ_t falls below 2%. Note that the true value is 1.1183.

Bisection Method

i	x_l	x_u	x_r	ϵ_t	$f(x_l)$	$f(x_r)$
1	0.8	1.6	1.2	7.3057	0.2425	-0.0748
2	0.8	1.2	1	10.5786	0.2425	0.1
3	1	1.2	1.1	1.6364		

(1) $X_L = 0.8 \quad X_U = 1.6$

$X_r = \frac{X_L + X_U}{2} = \frac{0.8 + 1.6}{2} = 1.2$

$\varepsilon_t = \frac{|1.1183 - 1.2|}{1.1183} \times 100 = 7.3057\%$

(2) $X_L = 0.8 \quad X_U = 1.2$

$X_r = \frac{0.8 + 1.2}{2} = 1$

$\varepsilon_t = \frac{1.1183 - 1}{1.1183} \times 100 = 10.5786\%$

(3) $\varepsilon_t = \frac{1.1183 - 1.1}{1.1183} \times 100 = 1.6364\%$

root = 1.1

False position method

The False-Position Method

Although bisection is a perfectly valid technique for determining roots, its approach is relatively inefficient. False position is an alternative based on a graphical insight. The root is given by:

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

Question 1: Use the bisection and false-position methods to determine the real root of the function

$$f(x) = 0.1x^2 - x \ln x$$

in the interval $[0.8, 1.6]$. Iterate until the estimated error ϵ_t falls below 2%. Note that the true value is 1.1183.

$x_L = 0.8 \qquad x_u = 1.6$

$f(x_u) = 0.1(1.6)^2 - 1.6\ln(1.6) = -0.496$

$f(x_L) = 0.1(0.8)^2 - 0.8\ln(0.8) = 0.2425$

$$x_r = 1.6 - \frac{-0.496(0.8-1.6)}{0.2425-(-0.496)} = 0.0483$$

False-Position Method

i	x_l	x_u	x_r	ϵ_t	$f(x_l)$	$f(x_r)$
1	0.8	1.6	1.0627	4.9715	0.2425	0.0483
2	1.0627	1.6	1.1103	0.7077		

$f(x_r)$ ← x_u

(1) $\varepsilon = \frac{1.1183 - 1.0627}{1.1183} * 100 = 4.9715\%$

$X_L = 1.0627 \qquad X_U = 1.6$

$f(X_L) = 0.1(1.0627)^2 - 1.0627 \ln 1.0627$

$= 0.0483$

$f(X_U) = -0.496$

$$X_r = 1.6 - \frac{-0.496(1.0627 - 1.6)}{0.0483 - (-0.496)}$$

$X_r = 1.1103$

$$\varepsilon_t = \frac{1.1183 - 1.1103}{1.1183} * 100$$

$= 0.707742\ \%$

root $X_r = 1.1103$