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EE 341

For KCST

EE101 - EE112 - EE211 - EE201 - EE212 - EE312 - EE341 - EE342 - EE351 - EE352 - EE365 - EE410 - EE441 - EE442 - EE443 - EE451

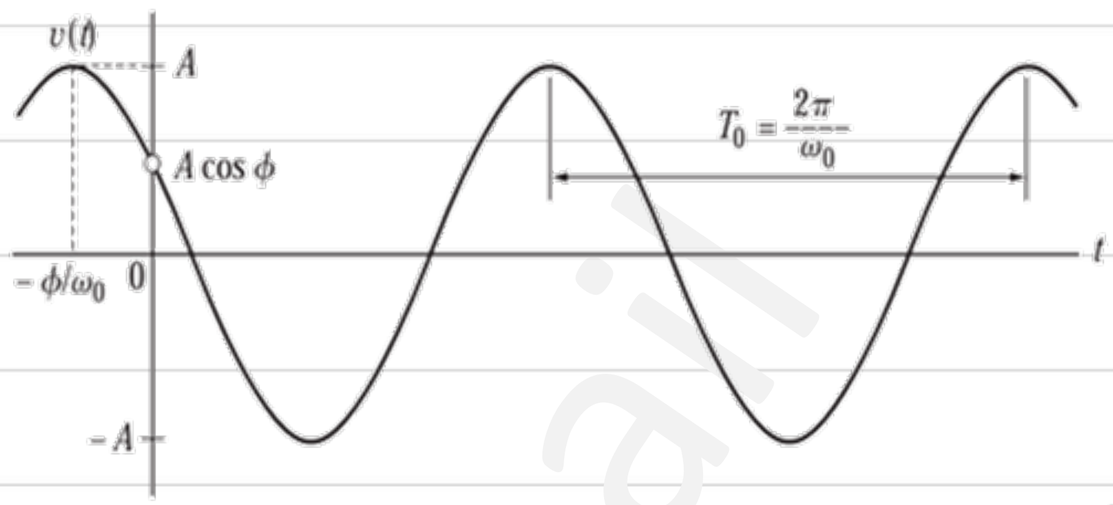
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<< Ch 2 - Part 1>>

Phasors and Line spectra:



$$v(t) = A\cos(\omega_0 t + \phi)$$

$\omega_0 = 2\pi f_0 \longrightarrow$ frequency in radians/sec.

$T_0 = \frac{1}{f_0} \longrightarrow$ period in sec.

$f_0 \longrightarrow$ cyclical frequency in Hz.

$\phi \longrightarrow$ Phase in radians.

Phasor Representation:

Euler's formula:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$j = \sqrt{-1} \quad , \quad j^2 = -1$$

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Excersie (1): use Euler's formula to prove the following

$$\cos(x) = \mathrm{Re}\{e^{jx}\} = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$

$$\sin(x) = \mathrm{Im}\{e^{jx}\} = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

$e^{jx} = \cos x + j\sin x \longrightarrow$ eq 1

$e^{-jx} = \cos x - j\sin x \longrightarrow$ eq 2

add eq 1 and 2

$e^{jx} + e^{-jx} = 2\cos x \qquad \times \frac{1}{2}$

$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$

Subtract eq 1 and 2

$e^{jx} - e^{-jx} = 2j\sin x \qquad \times \frac{1}{2j}$

$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$

Excersie (2)

Use the Euler's formula to show that $\frac{d}{dx}\sin x = \cos x$

$$\frac{d}{dx}\sin x = \frac{d}{dx}\left(\frac{1}{2j}\left(e^{jx} - e^{-jx}\right)\right) = \frac{1}{2j}\frac{d}{dx}\left(e^{jx} - e^{-jx}\right)$$

$$= \frac{1}{2j}\left[\frac{d}{dx}e^{jx} - \frac{d}{dx}e^{-jx}\right] = \frac{1}{2j}\left[je^{jx} + je^{-jx}\right]$$

$$= \frac{1}{2j} \times j\left[e^{jx} + e^{-jx}\right] = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] = \cos x$$

Excersie (3)

Use the Euler's formula to show that $\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$.

$$\cos^2 x = \left[\frac{1}{2}\left(e^{jx} + e^{-jx}\right)\right]^2$$

$$= \left(\frac{1}{2}\right)^2 \left(e^{jx} + e^{-jx}\right)^2$$

$$= \frac{1}{4}\left(e^{j2x} + e^{-j2x} + 2e^{jx}e^{-jx}\right)$$

$$= \frac{1}{4}\left(2 + e^{j2x} + e^{-j2x}\right)$$

$$= \frac{1}{4} \times 2\left[1 + \frac{1}{2}\left(e^{j2x} + e^{-j2x}\right)\right]$$

$$\cos^2 x = \frac{1}{2}\left[1 + \cos(2x)\right]$$

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<< Important Formulas:

* $\cos(-x) = \cos(x)$

* $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$

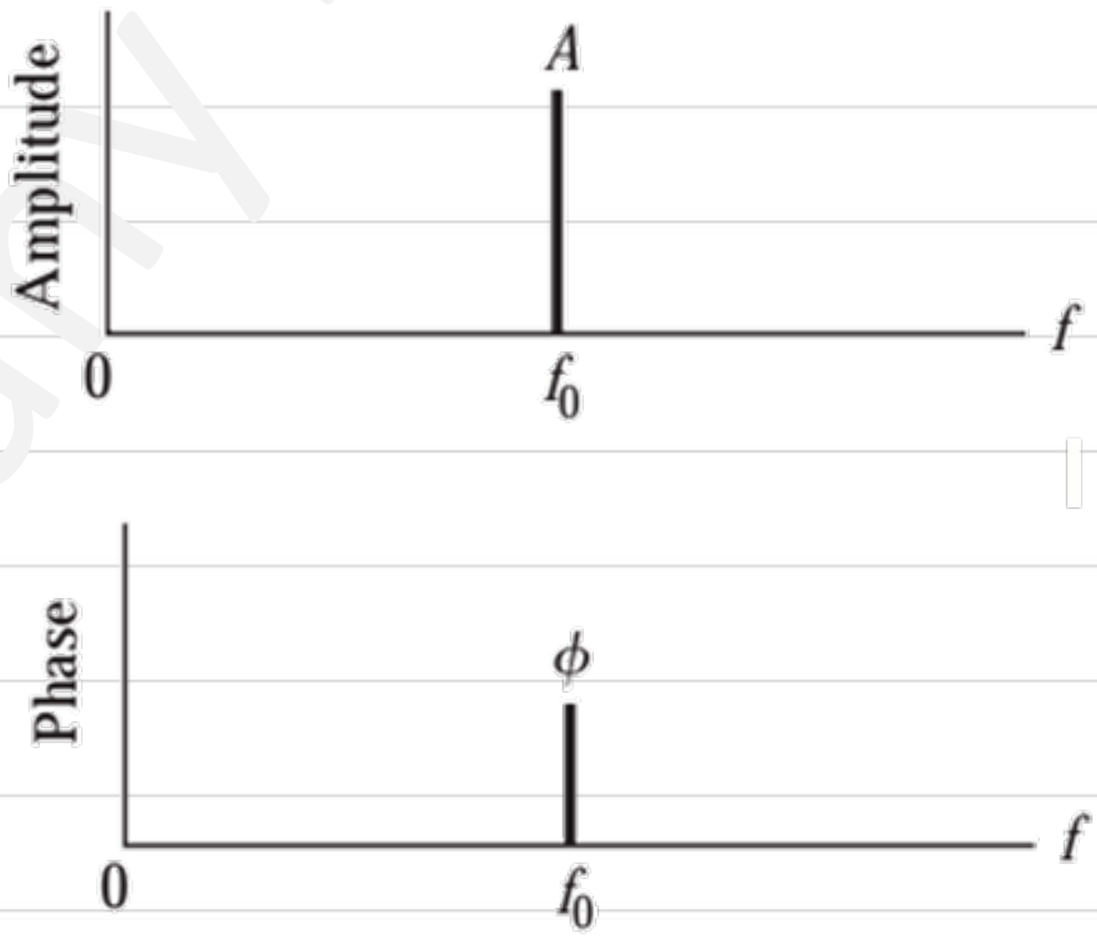
* $\sin^2 x = \frac{1}{2}\left(1 - \cos(2x)\right)$

* $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$

* $\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$

<< Phasors in Frequency-Domain:

line spectrum ↴



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Example:

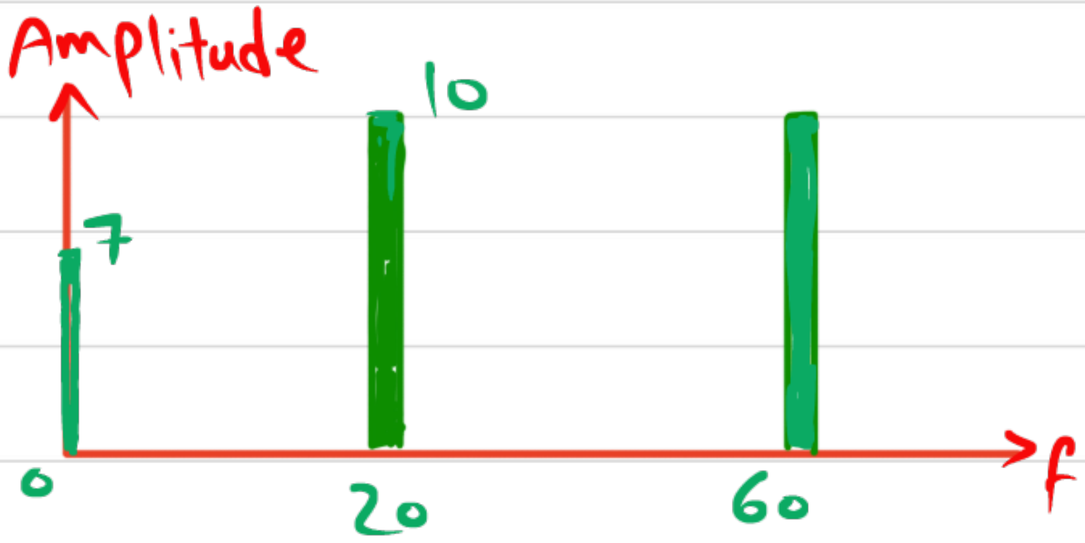
- Show the line spectrum of the following signal

$$w(t) = 7 - 10\cos(40\pi t - 60^\circ) + 4\sin 120\pi t$$

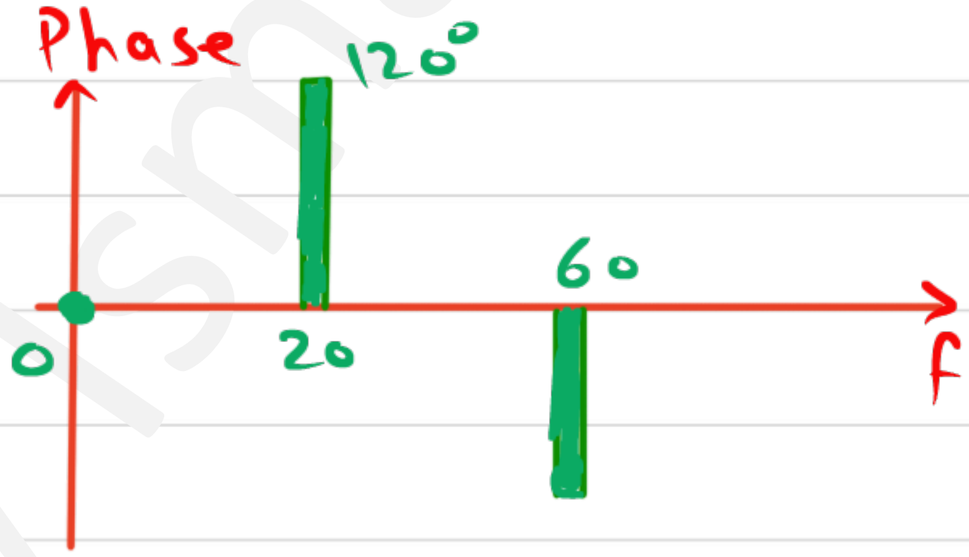
$w(t) = 7\cos(2\pi 0t) - 10\cos(2\pi 20t - 60^\circ) + 4\sin(2\pi 60t)$

$w(t) = 7\cos(2\pi 0t) + 10\cos(2\pi 20t + 120^\circ) + 4\cos(2\pi 60t - 90^\circ)$

Amplitude



Phase



Two-Sided Line spectra:

Amplitude

