Ma101-ch1

27 September 2025 13:39

Review of functions

Definition

A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

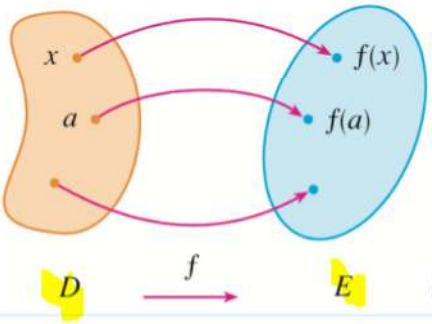
The set D is called **domain** of the function (the set of all real numbers for which the expression is defined).

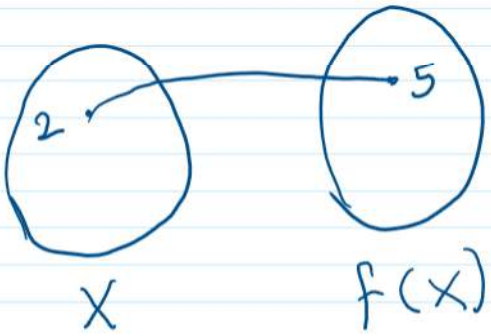
The **range** of f is the set E of all possible values of $f(x)$ as x varies throughout the domain.

$f(x) = x^2 + 1$

$x = 2$

$f(2) = 2^2 + 1 = 5$



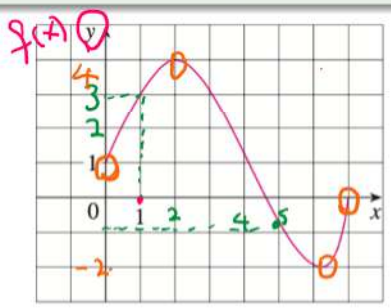


Example 1

The graph of a function f is shown below.

(a) Find the value of $f(1)$ and $f(5)$.

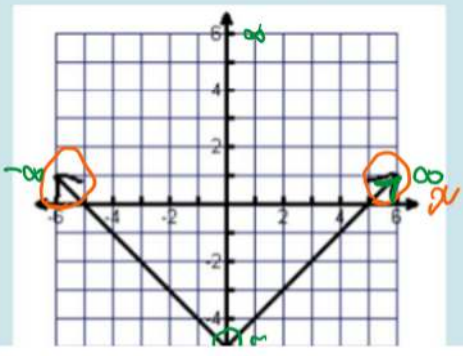
(b) What are the domain and range of f.

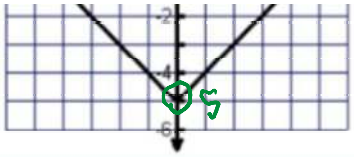


(a) $f(1) = 3$ and $f(5) \approx -0.7$.

(b) $D = [0, 7]$ and range $= [-2, 4]$.

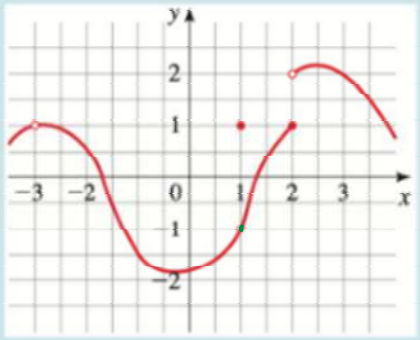
The Domain and the range of the function below are:





a. $Domain = [0, \infty)\ Range = [-5, \infty)$
b. $Domain = [-5, \infty)\ Range = (-\infty, \infty)$
c. $Domain = (-\infty, \infty)\ Range = [-5, \infty)$ ✓
d. $Domain = [-5, \infty)\ Range = [0, \infty)$

The graph of a function is given below



$\lim_{x \to 1^+} f(x) =$

a. 1
b. 0
c. -1 ✓
d. -2

functions أنواع

linear function ← (1) $f(x) = 2x^1 + 3$ deg=1, Constant term=3

quadratic ← (2) $f(x) = x^2 + 3x - 1$ deg=2, Con=-1

power ← (3) $f(x) = x^4 + 2x$ deg=4, constant=0

(4) $f(x) = x^5 + 2x + 2$ deg=5, Constant=2

] bionomial functions

deg = أكبر أس Constant term = x بدون

Exponential function:

$f(x) = a^x$

$f(x) = 3^x$

$f(x) = a$

$f(x) = 3^x$

$f(x) = 5^x$

logarithms function :

$$f(x) = \log_{\ln}(x+1)$$

rational function :

$$f(x) = \frac{P(x)}{q(x)}$$ P(x) ← numerator بسط ، q(x) ← denomerator مخرج

root function :

$$f(x) = \sqrt{x+1}$$

trigonometric function: الدوال المثلثية

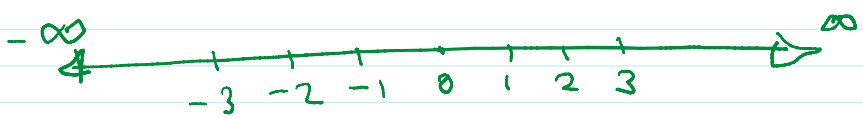
$f(x) = \sin, \cos, \tan$

Domain

جميع قيم x التي يمكن التعويض بها في f(x)

$$\text{Domain} = R = (-\infty, \infty)$$

all reals number



Except : ما عدا

1. Rational functions = $\frac{\text{بسط}}{\text{مقام}}$
2. root " = $\sqrt{\quad}$
3. logarithms " = $\log_{\ln}$

1. Rational function:

$$f(x) = \frac{P(x)}{Q(x)} \quad \begin{matrix}\text{بسط} \\ \text{مقام}\end{matrix}$$

Domain = R / {Zeros of denomenator}

أصفار المقام (أي قيم x التي تجعل المقام = 0)

Examples :

$$f(x) = \frac{1}{x}$$

Domain = $R/\{0\}$

$$f(x) = \frac{x}{x-2}$$

$$x - 2 = 0$$

$$\boxed{x = 2}$$

Domain = $R/\{2\}$

$$f(x) = \frac{x}{x^2 - 4}$$

$$f(x) = \frac{x}{x^2 - 4}$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x-2=0 \qquad x+2=0$$

$$x=2 \qquad x=-2$$

$$f(x) = \frac{x^2}{x^2 - 2x - 3}$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3=0 \qquad x+1=0$$

$$x=3 \qquad x=-1$$

$$\text{domain} = \mathbb{R} \setminus \{-1, 3\}$$

$$f(x) = \frac{x}{x^2+1}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$\text{domain} = \mathbb{R}$$

(2) root function:

$$f(x) = \sqrt{\quad}$$

$$\geq 0$$

* $f(x) = \sqrt{x-2}$

$x - 2 \geq 0$

$x \geq 2$



domain $= [2, \infty)$

* $f(x) = \sqrt{2x-8}$

$2x - 8 \geq 0$

$\frac{2}{2}x \geq \frac{8}{2}$

$x \geq 4$

domain $= [4, \infty)$

* $f(x) = \sqrt{3-x}$

$3 - x \geq 0$

$\frac{-x}{-1} \geq \frac{-3}{-1}$

$x \leq 3$

domain $= (-\infty, 3]$

في حالة يستخدم القوس
في سالب
نعكسه للجهة الثانية

$f(x) = \frac{2}{\sqrt{x-3}}$

لأنه في مقام

$$x - 3 > 0$$

$$x > 3$$

$$\text{domain} = (3, \infty)$$

$$f(x) = \sqrt{x^2 - 4}$$

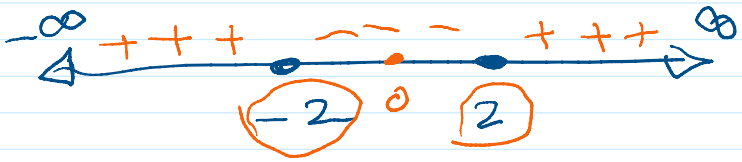
لازم الجذر التربيعي يكون أكبر من الصفر

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x - 2 = 0 \qquad x + 2 = 0$$

$$x = 2 \qquad x = -2$$



$$\text{domain} = (-\infty, -2] \cup [2, \infty)$$

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③ logarithms function:

$$f(x) = \log_{\ln}(\quad) \quad \text{(argument)} > 0$$

Ex

$$f(x) = \log(x-3)$$

$$x - 3 > 0$$

$$x > 3$$

$$\text{domain} = (3, \infty)$$

$$f(x) = \log(4-2x)$$

$$4-2x > 0$$

$$\frac{-2x}{-2} > \frac{-4}{-2}$$

$$x < 2$$

$$\text{domain} = (-\infty, 2)$$

Ex $\quad f(x) = \dfrac{\sqrt{x-1}}{x-2}$

$x-1 \geq 0$ | $x-2=0$ (cut)

$x \geq 1$ | $x=2$

$$\text{Domain} = [1,\infty) \setminus \{2\}$$

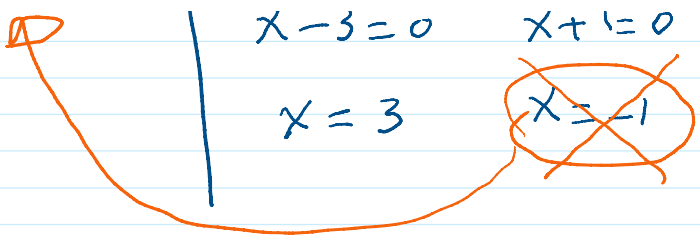
$$f(x) = \frac{\sqrt{x-1}}{x^2-2x-3}$$

$x-1 \geq 0$ | $x^2-2x-3=0$

$x \geq 1$ | $(x-3)(x+1)=0$

$x-3=0$ | $x+1=0$

$x=3$ | $x=-1$ (cut)



$$domain = [1, \infty) / \{3\}$$

logarithms function

$y = \log x$

base = 10 (اذا لم يكتب يكون)

$y = \ln x$

$e = 2.718$

natural logarithms

Shift ln = e

log , ln خواص

① $\log a + \log b = \log ab$

$\log 2 + \log 4 = \log 8$

② $\log a - \log b = \log \frac{a}{b}$

$\log 6 - \log 2 = \log 3$

③ $\log a^b = b \log a$

Solving equations

(1) $3^{2x+1} = 9$

$\log 3^{2x+1} = \log 9$

$(2x+1)\dfrac{\log 3}{\log 3} = \dfrac{\log 9}{\log 3}$

$2x + 1 = 2$

$\dfrac{2x}{2} = 2 - 1 = \dfrac{1}{2}$

$x = \dfrac{1}{2}$

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(2) $e^{x+1} = 4$

$\ln e^{x+1} = \ln 4$

$(x+1)\ln e = \ln 4$

$x = \ln(4) - 1$

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(2) $\log_3 2x = 2$

$2x = 3^2$

$$\frac{2}{2}x = \frac{9}{2}$$

$$x = 4.5$$

Solve $\ln_e(3x-1) = 2$

$$3x - 1 = e^2$$

$$\frac{3}{3}x = \frac{e^2+1}{3}$$

$$x = \frac{e^2+1}{3}$$



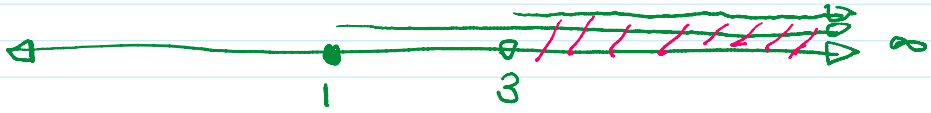
Review

find domain

$$f(x) = \sqrt{x-1} + \log(x-3)$$

$x - 1 \geq 0$ $\qquad$ $x - 3 > 0$

$x \geq 1$ $\qquad$ $x > 3$



domain $= (3, \infty)$

$$f(x) = \sqrt{2-x} + \ln(x-1)$$

$2 - x \geq 0$ $\qquad$ $x - 1 > 0$

$-x \geq -2$ $\qquad$ $x > 1$

$x \leq 2$



domain = (1, 2]

Solve

$$\log_2 x + \log_2 (x-2) = 3$$

$$\log_2 x(x-2) = 3$$

$$x^2 - 2x = 2^3$$

$$x^2 - 2x = 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$x - 4 = 0$ $\quad$ $x + 2 = 0$

$x = 4$ $\quad$ ~~$x = -2$~~

$$\boxed{x = 4}$$