Ma102-intgration

29 September 2025 11:50

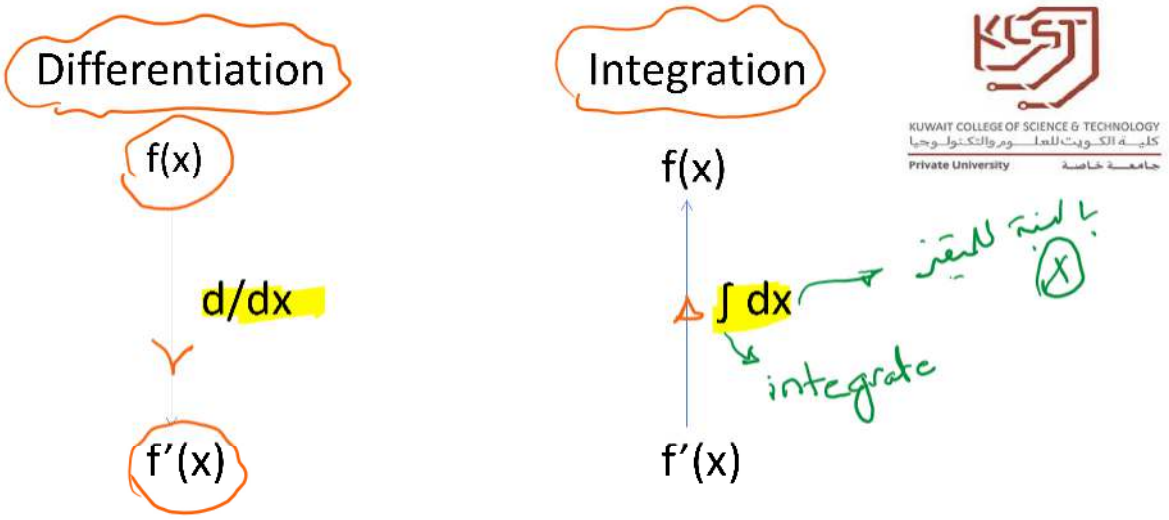
1-Review of Intergration

Fundamental Theorem of Integral and differential calculus

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The process of Integration is the inverse of that of differentiation

$$\int f(x)dx = F(x) \qquad \textit{Where} \qquad F'(x) = f(x)$$



So let's look back at the basic differentiation rules

Basic Differentiation rules

If $y = f(x) = C$, then $\frac{dy}{dx} = 0$

If $y = f(x) = x^n$, then $\frac{dy}{dx} = n x^{n-1}$

$y = x^5 \quad \frac{dy}{dx} = 5x^4$

If $y = f(x) = k \cdot u(x)$, then $\frac{dy}{dx} = k \cdot u'(x)$

$y = 5x^3 \quad \frac{dy}{dx} = 5 \cdot 3x^2 = 15x^2$

If $y = f(x) = u(x) \pm v(x)$, then $\frac{dy}{dx} = u'(x) \pm v'(x)$

$y = x^2 + x^3 \quad \frac{dy}{dx} = 2x + 3x^2$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}e^x = e^x$$

$y = e^{5x} \quad \frac{dy}{dx} = 5e^{5x}$

$$\frac{d}{dx}b^x = b^x \ln b$$

$y = 5^x \quad y' = 5^x \ln 5$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$y = \ln(x^2+1) \quad y' = \frac{2x}{x^2+1}$

$$\frac{d}{dx}\log_b x = \frac{1}{x \ln b}$$

$y = \log_3 x \quad y' = \frac{1}{x \ln 3}$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$y = \cos 5x \quad y' = -5\sin 5x$

$y = \sec 3x \quad y' = 3\sec 3x \tan 3x$

Differentiate/Find the Derivative of:

$$\frac{d}{dx}(5x^{-7} - 2x^{-4} + 5x + 2)$$

$= -35x^{-8} + 8x^{-5} + 5$

$$\frac{d}{dx}\left(\frac{e^x}{2} + \cos x - \sin x + \log_5 x + \tan x\right)$$

$= \frac{e^x}{2} - \sin x - \cos x + \frac{1}{x \ln 5} + \sec^2 x$

$$\frac{d}{dt}\left(\frac{5}{t^{1/5}}\right) \quad \frac{d}{dt}\left(5t^{\frac{-1}{5}}\right)$$

$= -1t^{\frac{-6}{5}}$

$$\frac{d}{dx}(2^x - 3^x + \csc x - \cot x)$$

$= 2^x \ln 2 - 3^x \ln 3 - \csc x \cot x + \csc^2 x$

$$y = (3x+2)(4x-5)$$ Product

$u \quad v$

$y' = u'v + uv'$

$y' = 3(4x-5) + (3x+2)4$

$= 12x - 15 + 12x + 8 = 24x - 7$

$$f(w) = \frac{3w+5}{w^2-3}$$ Quotient

$u \quad v$

$f'(x) = \frac{u'v - uv'}{v^2}$

$f'(x) = \frac{3(w^2-3) - (3w+5)*2w}{(w^2-3)^2}$

Derivative of a composite function-Chain Rule

If $y = f(u)$ and $u = g(x)$ define the composite function $y = f(u) = f[g(x)]$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \quad \text{provided } \frac{dy}{du} \text{ and } \frac{du}{dx} \text{ exist.}$$

Derivative of a composite function-Chain Rule

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Example

Differentiate $y = e^{2x^2}$

$y = e^U$ $\quad where\ U = 2x^2$

$$=> y' = \frac{dy}{du} \cdot \frac{du}{dx} = e^{2x^2} . 4x = 4xe^{2x^2}$$

Differentiate/Find the Derivative of:

$\frac{d}{dx} cos9x$

$= -9 \sin 9x$

$\frac{d}{dx} \ln(x^2 + 5)$

$= \frac{2x}{x^2+5}$

Integration

$$\int f(x)dx = F(x) \qquad \text{Where} \qquad F'(x) = f(x)$$

Table of Indefinite Integrals

1) $\int k\,dx = kx + C$

$\int 5\,dx = 5x + C$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

$\int x^3\,dx = \frac{x^4}{4} + C$

3) $\int \frac{1}{x}\,dx = \ln|x| + C$

4) $\int e^x\,dx = e^x + C$

$\int e^{2x}\,dx = \frac{e^{2x}}{2} + C$

5) $\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int 3^x\,dx = \frac{3^x}{\ln 3} + C$

6) $\int \sin x\,dx = -\cos x + C$

7) $\int \sec^2 x\,dx = \tan x + C$

8) $\int \cos x\,dx = \sin x + C$

9) $\int \csc^2 x\,dx = -\cot x + C$

10) $\int \sec x \tan x\,dx = \sec x + C$

11) $\int \csc x \cot x\,dx = -\csc x + C$

12) $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$

13) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

14) $\int -\frac{1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + C$

15) $\int \tan x\,dx = \ln|\sec x| + C$

16) $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

17) $\int \cot x = \ln|\sin x| + C$

18) $\int \csc x = \ln|\csc x - \cot x| + C$

Integrate/Find the integral/Evaluate the integral:

$$\int (x^7 - 6x + 8 - \frac{5}{x^2})dx = \int x^7 - 6x + 8 - 5x^{-2}\,dx$$

$$= \frac{x^8}{8} - \frac{6x^2}{2} + 8x - \frac{5x^{-1}}{-1} + C$$

$$= \frac{x^8}{8} - 3x^2 + 8x + 5x^{-1} + C$$

$$\int (x^{-4} + 2x^1 + \sin x + e^x)dx$$

$$= \frac{x^{-3}}{-3} + \frac{2x^2}{2} - \cos x + e^x + C$$

$$\int (\cos y - \sec^2 y)\,dx$$

$$= \sin y - \tan y + C$$

Definite Integrals

Definition: A definite integral is defined by

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

where F is any antiderivative of f, that is $F' = f$

Examples:

$$\int_3^6 \frac{1}{x}dx = [\ln x]_3^6 = \ln 6 - \ln 3 = \ln\frac{6}{3} = \ln 2$$

$$\int_1^2 (2x - 5)\,dx = 2\int_1^2 x\,dx - \int_1^2 5\,dx = [x^2]_1^2 - [5x]_1^2$$

$$= (4 - 1) - (10 - 5) = -2$$

Integration by Substitution

$$\int f(g(x))\, g'(x)\, dx$$

$$u = g(x)$$
$$du = g'(x)dx$$

The integral becomes $\int f(u)du$

Which could be then solved using the integration tables given before.

U-substitution for Chain rule

$$\int \cos(\pi x)\, \pi\, dx = ?$$

$$Let \quad U = \pi x \qquad Then \qquad \frac{du}{dx} = \pi \qquad And \qquad du = \pi dx$$

$$\int \cos(\pi x)\pi dx = \int \cos(U)dU = \sin(U) + c = \sin(\pi x) + c$$

$\int (3x^2)(x^3+1)^5 dx$

$u = x^3 + 1$

$du = 3x^2 dx$

$\int u^5 du = \frac{u^6}{6} + C$

$= \frac{(x^3+1)^6}{6} + C$

Table of Integrals

We can generalize the following rules

Function	*Integral*
$e^{kx} \quad (k \neq 0)$	$\frac{e^{kx}}{k} + c$
$\sin kx \quad (k \neq 0)$	$-\frac{\cos kx}{k} + c$
$\cos kx \quad (k \neq 0)$	$\frac{\sin kx}{k} + c$
$\sec^2 kx \quad (k \neq 0)$	$\frac{\tan kx}{k} + c$

Example $\int e^{4x}\,dx = \frac{e^{4x}}{4} + c$

Evaluate

$\int \sec^2 7x\,dx$

$= \frac{\tan 7x}{7} + c$

$\int \sin(\pi x) - 3\sin(3x)dx$

$= \frac{-\cos \pi x}{\pi} + \frac{3\cos 3x}{3} + c$

$\int x^3 \cos(x^4 + 2)\,dx$

$u = x^4 + 2$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + c$

$= \frac{1}{4}\sin(x^4 + 2) + c$

$\int \frac{4x^3 + 3x^2}{x^4 + x^3}dx$

$u = x^4 + x^3$

$du = (4x^3 + 3x^2)dx$

$\int \frac{du}{u} = \int \frac{1}{u}du$

$= \ln|u| + c$

$= \ln|x^4 + x^3| + c$